Influence of Multiplicative Noise on Electroconvections in Liquid Crystals

Jong-Hoon Huh

Department of Mechanical Information Science and Technology, Faculty of Computer Science and Systems Engineering, Kyushu Institute of Technology, Fukuoka 820-8502, Japan

huh@mse.kyutech.ac.jp
Abstract

We report noise-induced threshold shifts and pattern changes in ac-driven electrconvection (EC) in nematic liquid crystals. By controlling the noise intensity $V_N$ and cutoff frequency $f_c$, we investigate EC-threshold shifts and pattern structure. There exists a characteristic cutoff frequency $f_c^*$. For $f_c > f_c^*$, noise contributes to stabilizing EC (i.e., upward threshold shift), whereas for $f_c < f_c^*$, noise contributes to destabilizing EC (i.e., downward threshold shift). For $f_c = f_c^*$, noise makes no contribution to the threshold shift (i.e., neutral to the onset of EC). Moreover, the pattern structure of EC is changed by $V_N$, independent of $f_c$.

Keywords - electrohydrodynamic convection, nematic liquid crystals, noise
Introduction (1)

- Macroscopic structures in nonlinear dissipative systems

Thermodynamic Equilibrium System

A dissipative system is a thermodynamically open system which is operating out of, and often far from, thermodynamic equilibrium in an environment with which it exchanges energy and matter.

Examples

- in thermal convection
- in BZ reaction

Patterns or Rhythm (Dissipative structures)

Nonlinear Dissipative System

energy or matter
Introduction (2)

- Investigation of dissipative structures in electroconvection system

\[ V > V_c \]

\[ V < V_c \]

\[ d \]

\[ f_{cd} \]

Voltage [V]

Frequency [Hz]

MBBA (4'-methoxy-benzilidene-4-n-buthy-aniline)

Various electrohydrodynamic instabilities (i.e., dissipative structures in nematic liquid crystals.)
**Our objectives**

**In the presence of external multiplicative noise:**

1. What is the influence of noise on thresholds for Ecs (WD and CV)?
2. What is the influence of noise on pattern structures (WD and CV)?

By controlling the noise intensity $V_N$ and cutoff frequency $f_c$.

- To investigate an expanded Carr-Helfrich mechanism considering noise.

$$V_c = V_c(V_N, f_c)$$

- To examine characteristic wavelengths $\lambda_{WD}$ for WD and $\lambda_1$ and $\lambda_2$ for CV.
By applying an additional noise (with $V_N$ and $f_c$) to ac-driven ECs (determined with $V$ and $f$)

**Experimental setup**

**Sample cell**

**Colored noise**

$$V_N = \sqrt{\langle V_p^2 \xi(t)^2 \rangle}$$
Results (1)

In the absence of noise;

By Carr-Helfrich Mechanism

○ Conductive regime ($f < f_{cd}$ for Williams domains: WD)

$$V_{c0}^2(f) = \frac{V_0(1 + 4\pi^2 f^2 \tau_{\sigma}^{-2})}{\xi^2 - (1 + 4\pi^2 f^2 \tau_{\sigma}^{-2})}$$

$$\tau_{\sigma} = \frac{\epsilon_0 \epsilon_1}{\sigma_\perp} : \text{charge relaxation time}$$

$$\xi^2 = \left(1 - \frac{\sigma_\perp}{\epsilon_\parallel \epsilon_\perp} \right) \left[1 + \frac{\alpha_\parallel \epsilon_\perp}{\gamma_1 \epsilon_\perp} \right] : \text{Helfrich parameter}$$

○ Dielectric regime ($f > f_{cd}$ for chevrons: CV)

$$V_{c0}^2(f) = \frac{190 \gamma_1 f}{(\epsilon_\parallel / \epsilon_\perp - \sigma_\perp / \sigma_\parallel) \epsilon_\parallel}$$

In the presence of noise ($V_{NB} f_c$);

By Expanded Carr-Helfrich mechanism

$$q + \frac{q(t)}{\tau} + \sigma_\parallel \left[ E_\parallel \sin\omega t + \xi(t) \right] \psi = 0$$

$$\dot{\psi} + \{E_\parallel \sin\omega t + \xi(t)\} \frac{q(t)}{\eta} + \lambda \left[A + \{E_\parallel \sin\omega t + \xi(t)\}^2\right] \psi(t) = 0$$
Results (2)

- Conductive regime (f < f_{cd} for Williams domains: WD)

\[ V_c^2 = V_{c0}^2 + b V_N^2 \]

- \( f > f^* \): upward shift in threshold (\( b > 0 \))
- \( f < f^* \): downward shift in threshold (\( b < 0 \))
- \( f = f^* \): neutral noise on threshold (\( b = 0 \))

The neutral noise (with \( f = f^* \) for \( b = 0 \)) is independent of ac frequency \( f (< f_{cd}) \).

Noise plays a role not only in suppressing ECs but also in destabilizing them.
Results (3)

- Conductive regime (f < f_{cd} for Williams domains: WD)

The relationship $f'_c (f_{cd})$ between the characteristic external ($1/f'_c$) and internal ($1/f_{cd}$) time scales is found, which gives us a crucial hint to understand the influence of noise.

The characteristic wavelength $\lambda_{WD}$ is independent of cutoff frequency $f_c$, which monotonically decreases with noise intensity $V_N$.

$\tilde{f} = 0.1 f_{cd}^{1.4}$

Stabilization effect of high-frequency components ($> f_{cd}$) and destabilization effect of low frequency ones ($< f_{cd}$) are cancel out for $f_c = f'_c$. 

The relationship $f'_c (f_{cd})$ is found, which gives us a crucial hint to understand the influence of noise.
Results (4)

- Dielectric regime ($f > f_{cd}$ for chevrons: CV)

An interesting influence of noise on the threshold of EHC is found, which is reminiscent of stochastic resonance.

As contrasted with WD, the characteristic wavelengths $\lambda_1$ and $\lambda_2$ smoothly increase with noise intensity $V_N$. 
Discussion and Conclusion

What is the influence of noise on threshold for ECs?
What is the influence of noise on pattern structures?

What we have known:
- Colored noise contributes to the onset of each EC depending on its cutoff frequency.
- Depending on colorization level (fc), stabilization and destabilization effects were found, which are determined internal timescale (1/fcd) for EC.
- Noise intensity plays a role in pattern formation with respect to each mode.
- However, the behaviors of respective modes are quite distinguishable.

What we have unknown:
- Detailed mechanisms of the response of ECs to noise for respective modes (especially, CV)

What we are interested in:
- Influence of colored noise on other dissipative systems such as biosystem and nanotechnology, in which noise is unavoidable.
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Contact me for any questions.

Email: huh@mse.kyutech.ac.jp