Noise-induced threshold shift and pattern formation in electroconvection by controlling characteristic time scales

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We report external noise-induced threshold shift and pattern formation in ac-driven electroconvection (EC) in nematic liquid crystals. We investigate the threshold shift by controlling the characteristic cutoff frequency $f_{cd}$ in the absence of colored noise $V_N = 0$, and $b$ represents the sensitivity of the response of WDs to external noise. Previously reported upward threshold shifts ($b > 0$) can be intuitively understood by considering stabilization effects due to the noise-driven random oscillation against the periodic ac-based CH instability [5,9]. When $f_c$ (or $\tau_N$) is controlled, the slope $b$ varies for relatively small intensities ($V_N > 20$ V) [19,21]. Recently, we noticed that $b$ varies notably with the characteristic frequency $f_{cd}$ as well as $f_c$. For EC, $f_{cd}$ is known as the critical frequency and divides the conductive ($f < f_{cd}$) and dielectric ($f > f_{cd}$) regimes [25]. This $f_{cd}$ can dominate the shift problems in EC. Unfortunately, to the best of our knowledge, no study for $f_{cd}$-considered threshold shifts has been carried out, since most of the previous studies have considered a fixed $f_{cd}$ (due to using one sample cell) [5,16–21].

Here, $V_{cd}$ indicates the threshold voltage in the absence of noise ($V_N = 0$), and $b$ represents the sensitivity of the response of WDs to external noise. Previously reported upward threshold shifts ($b > 0$) can be intuitively understood by considering stabilization effects due to the noise-driven random oscillation against the periodic ac-based CH instability [5,9]. When $f_c$ (or $\tau_N$) is controlled, the slope $b$ varies for relatively small intensities ($V_N > 20$ V) [19,21]. Recently, we noticed that $b$ varies notably with the characteristic frequency $f_{cd}$ as well as $f_c$. For EC, $f_{cd}$ is known as the critical frequency and divides the conductive ($f < f_{cd}$) and dielectric ($f > f_{cd}$) regimes [25]. This $f_{cd}$ can dominate the shift problems in EC. Unfortunately, to the best of our knowledge, no study for $f_{cd}$-considered threshold shifts has been carried out, since most of the previous studies have considered a fixed $f_{cd}$ (due to using one sample cell) [5,16–21].

By considering an expanded CH mechanism in external multiplicative noise [5,16,20], a linear relationship can be derived as

$$V_c^2 = V_{cd}^2 + bV_N^2. \quad (1)$$

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In this Rapid Communication, we address an ac-driven electroconvection (EC) system [14,15] as a dissipative system having a controllable internal time scale, and investigate the threshold shifts [5,16–21] by controlling the external time scale of noise (i.e., using colored noise; $\tau_N \neq 0$) [22]. Under the conditions of both time scales, the present physical system shows completely different threshold shifts than are also expected in other dissipative systems such as biosystems [11–13,23] and chemical systems [24].

In an EC system, a nematic liquid crystal (NLC) sandwiched between two parallel electrodes is subjected to an external electric field. If the field is purely deterministic (e.g., a sinusoidal wave), a rich variety of stationary and nonstationary patterns are found [14]. A typical EC pattern, the Williams-Carr-Helfrich (CH) instability [15]. However, if an external stochastic noise is superposed on the field, the EC system is expected to show unique instability aspects.

To date, several groups have intensively investigated threshold shifts for WDs by controlling the noise intensity $V_N$ and correlation time $\tau_N$ (or cutoff frequency $f_c$) [5,16–21].
such a pure noise-induced EC was demonstrated at $V$ and $f_c$ having different electrical conductivities (determining $b$ value of 40 kHz), whereas for cell 3 (which has a much larger $f_{cd}$ and parameter and the charge relaxation time, respectively [15].)

The stochastic intensity $\langle VN \rangle$ measured in cell 2 (having $f_{cd}$ = 25 Hz) was fixed at $f = 30$ Hz ($< f_{cd}$ for all sample cells) to achieve WDs ($f < f_{cd}$), except for Fig. 4. We used several sample cells having different electrical conductivities (determining $f_{cd}$) in Table I. The electrical conductivity $\sigma$ ($= 10^{-5}$–$10^{-2}$ $\Omega^{-1}$ m$^{-1}$) was controlled by doping tetra-n-butyl-ammonium bromide (TBAB) into MBBA from 0 to 1.0 wt %. The details of the experiment were described in Ref. [21].

First, we investigate the $f$ dependence of $V_c$ in the absence of noise, to determine $f_{cd}$ for EC. According to the CH mechanism, $V_c \sim (1 + 4\pi\tau_f^2 t_r^2)/[\zeta^2 - (1 + 4\pi\tau_f^2 t_r^2)]$ for the conductive regime ($f < f_{cd}$) and $V_c \sim \sqrt{f}$ for the dielectric regime ($f > f_{cd}$) [25]. Here, $\zeta$ and $\tau_f$ indicate the Helfrich parameter and the charge relaxation time, respectively [15]. By considering the sharp variation of $V_c(f)$ around $f_{cd}$, we determined $f_{cd}$ for each cell, as described in Table I.

Next, we verify Eq. (1) using all sample cells having different $f_{cd}$ values (Table I). Figure 1 shows the behavior of $V_c(V_N)$ measured in cell 2 (having $f_{cd}$ = 2.5 kHz). The sensitivity of the response $b$ is positive for $f_c = 10$–200 kHz, $b = 0$ for $f_c = 5$ kHz, and $b$ is negative for $f_c = 200$ Hz to 2 kHz. Thus, the stabilization ($b > 0$) and destabilization ($b < 0$) effects on the onset of EC are switched at a characteristic cutoff frequency $f_c = f_c^*$ ($b = 0$). The colored noise having $f_c < f_c^*$ no longer functions as normal noise (to suppress the instability). Furthermore, pure noise-induced EC ($V_N \rightarrow 0$) is found for noise having $f_c > f_c^*$ ($b < 0$) [21]. An example of such a pure noise-induced EC was demonstrated at $V_N \sim 9$ V and $f_c = 1$ kHz. Voltage $V_N$ increases monotonically with increasing $f_c$, and diverges at $f_c^*$ [21].

On the other hand, for cell 1 (which has a relatively small $f_{cd}$ value of 160 Hz), $b$ was positive for all ranges of $f_c$ (200 Hz to 200 kHz), whereas for cell 3 (which has a much larger $f_{cd}$ value of 40 kHz), $b$ was negative for $f_c = 200$ Hz to 100 kHz, and $b \sim 0$ for $f_c > 200$ kHz. The $f_c^*$ ($b = 0$) for each cell crucially depends on $f_{cd}$.

The slope $b$ is plotted as a function of $f_c/f_c^*$, as shown in Fig. 2. It can be observed that at $f_c/f_c^* = 1$ ($b = 0$), the average slope $b(>0)$ increases gradually and remains constant with increasing $f_c$ (i.e., less colored noise). The external noise having $f_c/f_c^* > 10^2$ (with a saturated value $b \sim 0.8$) can be qualified as normal white noise ($f_c \rightarrow \infty$) for EC (e.g., $f_c > 10^3$ Hz for cell 2). Such noise having $f_c/f_c^* > 1$ (i.e., $b > 0$) contributes to stabilizing EC. Similarly, at $f_c/f_c^* = 1$ (b = 0), the average slope $b (< 0)$ decreases gradually with decreasing $f_c$ (i.e., more colored noise). Such noise having $f_c/f_c^* < 1$ (i.e., $b < 0$) contributes to destabilizing EC. Thus, the noise having $f_c/f_c^* = 1$ (i.e., $b = 0$) is called neutral
FIG. 3. Response of WD to colored noise (having \( f_c \)) with respect to the critical frequency \( f_{cd} \). The dotted line indicates the neutral noise (having \( f_c = f_{cd}^\ast \)) on \( V_c \) satisfying Eq. (2). See Fig. 1 and Table I.

Noise for the EC-onset threshold. This neutral noise does not contribute to the threshold shift.

Moreover, \( f_{cd}^\ast \) is expressed as a function of \( f_{cd} \), as shown in Fig. 3. By using the least-mean-square approximation, we find the power law

\[
f_{cd}^\ast = hf_{cd}^\alpha, \tag{2}
\]

where \( \alpha \approx 1.4 \) and \( h \approx 0.1 \) in our experiment. This indicates that the noise having an external characteristic time \( 1/f_{cd}^\ast \) satisfying Eq. (2) with respect to the intrinsic characteristic time \( f_{cd}^\ast \) is neutral to the onset of EC. In addition, we verified the validity of Eq. (2) by using two different cells: P25, which has a planar alignment with \( d = 25 \, \mu m \), and H50, which has a homeotropic alignment with \( d = 50 \, \mu m \). As shown in Fig. 3, Eq. (2) is still valid for these cells. In other words, the power law may be satisfied under conditions of different thickness \( d \) or different initial alignments \( n_0 \). As shown in Fig. 4, \( \alpha \) and \( h \) in Eq. (2) are independent of ac frequency \( f \) (i.e., constant \( f_{cd}^\ast \) without respect to \( f \) for each cell (having a fixed \( f_{cd} \))), whereas \( b \) in Eq. (1) strongly depends on \( f \) [16,21] and smoothly increases with increasing \( f \) (at fixed \( f_{cd} \)).

Finally, we examine the characteristic wavelength \( \lambda_{WD} \) for WDs by controlling \( V_N \) and \( f_c \). As shown in Fig. 5, \( \lambda_{WD} \) smoothly decreases with increasing \( V_N \) for \( f_c = f_{cd}^\ast \) as well as \( f_c \neq f_{cd}^\ast \). In other words, the structure of WDs is not affected by the degree of colorization of noise.

Multiplicative (quasiwhite) noise (\( f_c > f_{cd}^\ast \)) may contribute to stabilizing against the CH instability (i.e., spatially periodic charge focusing by the ac field) because it reduces the amount of space charge by random oscillation. As a result, \( V_c \) increases with increasing \( V_N \) (i.e., \( b > 0 \)). Such a stabilization effect of noise was well understood in earlier theories [5,16,20] and experiments [5,16,19,21]. However, sufficiently colored noise (\( f_c < f_{cd}^\ast \)) having a long correlation time \( \tau_N \) can accumulate more space charge for EC by aiding the ac field. As a result, \( V_c \) decreases with increasing \( V_N \) (i.e., \( b < 0 \)). For \( f_c = f_{cd}^\ast \), because the stabilization (\( b > 0 \)) and destabilization (\( b < 0 \)) cancel each other, the noise is neutral with respect to the onset of EC (\( b = 0 \)). However, the pattern structure \( \lambda_{WD} \) strongly depends on \( V_N \) but not on \( f_c \).

In conclusion, noise-induced threshold shifts in EC are determined by the intrinsic characteristic frequency \( f_{cd} \) for EC as well as by the cutoff frequency \( f_c \) and intensity \( V_N \) of the external noise. The response sensitivity \( b \) of EC to the noise is varied by changing \( f_c \) and \( f_{cd} \). The changeover \( (b = 0) \) between the stabilization and destabilization is expressed as a power law [Eq. (2)] in which a characteristic cutoff frequency...
$f^*_c(f_{cd})$ for the noise appears. The sufficiently colored noise having $f_c < f^*_c$ contributes to destabilizing the onset of EC $(b < 0)$, whereas noise having $f_c > f^*_c$ stabilizes the onset of EC $(b < 0)$ and no effect of stabilization is observed for noise having $f_c = f^*_c (b = 0)$. However, the pattern structure is affected only by $V_N$ (without relation to $f_c$). The relationship $f^*_c(f_{cd})$ between the characteristic external $(1/f^*_c)$ and internal $(f_{cd}^{-1})$ time scales (in Fig. 3) can directly explain how the difference of threshold shifts (i.e., $f_c < f^*_c$ having noise having $f_{cd}^{-1}$) time scales (in Fig. 3) can directly explain how the difference of threshold shifts (i.e., $b > 0$, $b < 0$, and $b = 0$) appeared in the previous studies [5,16–21]. As elucidated in the present EC systems, noise-induced phenomena such as stochastic resonance and synchronization may be systematically understood with considering both time scales. In this point of view, there exist a variety of applicable researches such as noise-mediated biorhythms having different levels of time scales (e.g., ultradian and circadian rhythms) [11], stochastic synchronization in brainwaves (e.g., alpha and delta waves) [12,13], and fluctuating-light effects on photosynthesis, including photochemical cycles (e.g., xanthophyll cycle) [23]. In these researches, the stochastic matching (i.e., applying adequate colored noises) with the intrinsic time scales of the systems may drastically change the response of the systems to noise.

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