Pure Noise-Induced Pattern Formations in a Nematic Liquid Crystal

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We report pure noise-induced pattern formations in a homeotropically aligned nematic liquid crystal. Williams domains and stripe patterns are induced by colored and white noises, respectively, and the Fredericks transition is also found, which has no dependence on correlation time of the external noise. Furthermore, threshold behavior in the presence of a regular (sinusoidal) field superposed with the noise was investigated to understand the intrinsic role of the noise on the mechanisms driving noise-induced patterns and transitions.

KEYWORDS: noise, electrohydrodynamic instability, liquid crystal, homeotropic alignment

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Noises usually play an important role in bifurcation phenomena in nonlinear nonequilibrium systems. Well-known and nontrivial noise phenomena are, e.g., stochastic resonance, multiplicative stochastic process, and noise-induced order (or phase transitions). Though they have been continuously studied for more than 30 years, no universal understanding has been obtained. This may be due to strong nonlinearity of related phenomena. In the above-mentioned studies, noise is an additional control parameter and always superposed with periodic signals that work as main ones. Now we have a question whether noise itself can be a control parameter dominating bifurcation phenomena and/or pattern formation. In order to make clear on this point, we concentrate to study pure noise effects on pattern formation processes using electrohydrodynamic convection (EHC) in nematic liquid crystals (NLCs).

EHC is a well-known and well-studied pattern formation phenomenon. There are two types of EHCs depending on the initial preparation of NLCs. In the planar alignment case the initial director \( n_0 = (1,0,0) \) in the standard coordinate system, one finds successive pattern formations [initial planar state \( \rightarrow \) Williams domain (WD) \( \rightarrow \) grid pattern \( \rightarrow \) dynamic scattering mode (DSM)] with increasing an AC field. In the homeotropic alignment case \( n_0 = (0,0,1) \); see Fig. 1(a), on the other hand, quite different evolution [initial homeotropic state \( \rightarrow \) Fredericks transition (FT) \( \rightarrow \) soft-mode turbulence (SMT)] is found.

In this letter we report pure noise-induced pattern formations found in the homeotropic case, which have not been observed in the planar one. When applying an external pure (electric) noise across a slab of a homeotropically aligned NLC, a variety of instabilities are found. Similarly to the AC or DC field case, we found a similar transition [i.e., initial homeotropic state \( \rightarrow \) FT \( \rightarrow \) EHC (e.g., SMT) with increasing the noise intensity [see Figs. 1(a)–1(c)]. In this study, our interest is mainly confined to FT and WD-like SMT very close to the onset of EHC, though complete SMT are induced at higher intensity. The phase diagram in the cutoff frequency and intensity of the noise was investigated. Furthermore, the threshold behavior in the presence of a regular AC field superposed with the noise was investigated to understand the intrinsic role of the external noise on the noise-induced pattern-formation mechanisms.

The noise field \( N(t) \) was applied across a slab of a NLC, MBBA (p-methoxybenzylidene-p'-n-butylaniline), sandwiched between two parallel transparent electrodes (indium tin oxide), as shown in Figs. 1(a)–1(c). The gap \( d \) between the electrode surfaces given with homeotropically aligning treatment [Fig. 1(a)] is 50 \( \mu \)m, and the lateral (active) size is 1 \( \times \) 1 cm\(^2\). The Gaussian-type noise \( N(t) \) was created by a wave-generating synthesizer (Hioki 7075), and amplified by an Amp (FLC Electronics). In particular, cutoff frequency \( f_c \)-dependent colored noise was generated from the low-pass filters of the synthesizer, which passes low-frequency signals but attenuates signals with frequencies higher than \( f_c \). In this study the intensity \( V_N = \sqrt{(N(t)^2) \cdot d} \) and \( f_c \) (or the correlation time \( \tau_N \)) of the noise are the control parameters. The patterns were observed in the \( xy\)-plane parallel to the electrodes by using a charge-coupled-device camera (Sony XC-75) mounted on a polarizing microscope (Meijitech ML9300). To capture and analyze the patterns on a computer, image processing software (Scion Image Beta 4.0.2) and an image board (Scion PCI-VE5) were used. The electric conductivities and dielectric constants for the present liquid crystal were \( \sigma_f = 5.03 \times 10^{-8} \Omega^{-1} \text{m}^{-1} \), \( \sigma_{\perp} = 4.16 \times 10^{-8} \Omega^{-1} \text{m}^{-1} \), and \( \varepsilon_{||} = 4.74 \), and \( \varepsilon_{\perp} = 5.52 \), respectively, at room temperature (\( T \approx 25^\circ \text{C} \)). Here the subscripts \( || \) and \( \perp \) represent orientations parallel and perpendicular to the director \( n \) of a NLC, respectively.

First, the phase diagram was investigated with varying \( V_N \) and \( f_c \) of noise. Figures 2(a)–2(d) show typical patterns observed in the system. Surprisingly, conventional patterns governed by a regular field in this system were also observed in the present noise field [see Fig. 1(d)]. With increasing \( V_N \), the initial homeotropic state is destabilized, and then the FT [Fig. 2(a)] is found at a characteristic intensity \( V_{N,FT} \approx 3.8 \text{ V} \) with no dependence on \( f_c \), as shown in Fig. 2(e). On the other hand, EHC patterns [Fig. 2(d)] and stripe patterns [SPs; Fig. 2(b)] are found to be secondary instabilities at their corresponding threshold intensities \( V_{N,EHC} \) and \( V_{N,SP} \), respectively. However, unlike \( V_{N,FT} \), they depend strongly on \( f_c \). As shown in Fig. 2(e), a certain characteristic
patterns with large patterns may be viewed as (sufficiently) colored-noise type instabilities. In the absence of the noise field ($V = V_0$), the AC-field thresholds are $V_{S,FT} \approx 4.0$ V and $V_{S,EHC} \approx 6.0$ V for a fixed AC frequency $f = 30$ Hz ($< f_c \approx 400$ Hz; a characteristic frequency for AC-driven EHC dividing conduction and dielectric regimes). These are predictable values for AC-driven instabilities from the conventional theory.\(^{13,14}\) Considering $V_{N,FT}$ ($V_S = 0$ V) nearly equivalent to $V_{S,FT}$ ($V_N = 0$ V), the FT can be induced only by the pure intensity of the applied field, independent of the type of electric field (AC, DC, or noise alone). Considering $V_{N,EHC}$ ($V_S = 0$ V) much larger than $V_{S,EHC}$ ($V_N = 0$ V), on the other hand, the noise-induced EHC should be explained by other characteristics of noise, such as $f_c$ (or $\tau_N$), as well as intensity $V_N$.

As shown in Fig. 3(b), the threshold $V_{S,FT}$ for FT monotonically decreases with increasing $V_N$. This behavior means that the destabilization effect of noise facilitates the Fredericks instability. On the other hand, the threshold $V_{S,EHC}$ ($V_N$) for EHC shows slightly complicated behavior. In particular, for relatively low noise intensity ($V_N \leq 8$ V), noise plays a role in stabilizing the state (i.e., the homoge-
Previously reported results. Also, the noise (e.g., temperature-dependent viscoelastic and/or electric properties (i.e., the slope \(c\) of noise effect depends on frequency \(f\) of the AC field as well as the characteristics of noise \(f_c\) or \(\tau_N\).\(^{21-23}\))

Finally, the threshold \(V_{S,\text{EHC}}(V_N)\) for EHC was measured in both (nearly) white \((\tau_N = 8.2 \mu\text{s}; f_c = 250 \text{kHz})\) and colored \((\tau_N = 278 \mu\text{s}; f_c = 500 \text{Hz})\) noises superposed with the AC field \((f = 30 \text{Hz})\), in order to compare with the previously reported results.\(^{16,17,19-23}\) Also, the \(V_{FT}(V_N)\) for the FT is plotted again in Fig. 3(c). In the case of white noise \((f_c = 250 \text{kHz})\), a linear relation (1) is obtained, as shown in Fig. 3(c).

\[
V^2_{S,\text{EHC}} = V^2_{S,\text{EOJ}} + aV^2_N \quad (1)
\]

\[
V^2_{S,\text{FT}} = V^2_{S,\text{FOJ}} + bV^2_N \quad (2)
\]

Here, \(V_{S,\text{EOJ}}\) and \(V_{S,\text{FOJ}}\) represent the thresholds for AC field-induced EHC and FT in the absence of noise \((V_N = 0 \text{V})\), respectively. In fact, the relation (1) is valid for large range of \(V_N (0 < V_N < 25 \text{ V})\).\(^{23}\) Obviously, this shows the threshold shift-up for EHC due to stabilization effect of the noise, which is good agreement with the previous results.\(^{16,17,19-23}\)

Since the homogeneous Fredericks state is suppressed by the noise, the threshold \(V_{S,\text{EHC}}\) increases monotonically with increasing \(V_N\) (i.e., the slope \(a = 0.627 > 0\)). In the case of colored noise \((f_c = 500 \text{ Hz})\), the linear relation (1) remains still valid for except low intensity of noise \([V_N < 1 \text{ V}; \text{see Fig. 3(b)}]\), but the slope \(a\) is negative \((a = -0.256)\). The sensitivity (i.e., the slope \(a\)) of noise effect depends on frequency \(f\) of the AC field as well as the characteristics of noise \((f_c\) or \(\tau_N\)).\(^{21-23}\)

On the other hand, the relation (2) for FT is also valid and \(b\) is always negative \((b = -1.18)\). The FT does not depend on cutoff frequency \(f_c\) (or \(\tau_N\)) of noise as well as frequency \(f\) of the AC field. The slope \(b \sim -1\) [see the inset in Fig. 3(c)] means that for FT the total intensity only plays a role in destabilizing the system. Although the threshold \(V_{S,\text{SP}}\) for SPs is not seen in Figs. 3(b) and 3(c), it also showed the joint effects of the superposed fields.

It is worth mentioning that the present pure noise-induced instabilities could be observed in the homeotropic alignment cell \([n_0 = (0, 0, 1)\]), but not in the more conventional planar case with different symmetry \([n_0 = (1, 0, 0)\]). In particular, considering AC-driven EHCs at a nearly equivalent intensity \((V_S = 6-7 \text{ V})\) in both alignments,\(^{14,15}\) it is inapplicable that pure noise-induced EHC patterns are not observable in the planar alignment. The present stable EHC patterns should be distinguished from on–off intermittent EHC patterns (driven by dichotomous noise) in the planar alignment cell.\(^{12}\)

Moreover, a certain characteristic cutoff frequency \(f_*^0\) (or \(\tau^*_N\)) for the appearance of EHC or SP in Fig. 2(e) varies remarkably with temperature. These observations indicate that the pure noise-induced pattern formation is very sensitive to the constraints of the system, such as the director alignment (i.e., anchoring force or symmetry conditions) and the material parameters of liquid crystals (e.g., temperature-dependent viscoelastic and/or electric properties), or to thermally internal noise conditions.\(^{24}\)

Fig. 3. Threshold behavior for FT and EHC in the case of noise \(N(t)\) superimposed on (sinusoidal) AC field \(S(t)\). (a) An applied field for this phase diagram, which was obtained from a combiner (AC frequency \(f = 30 \text{Hz}\) and noise cutoff frequency \(f_c = 500 \text{Hz}\) or 250 kHz). (b) Phase diagram in the noise intensity \((V_N)\) and AC-field intensity \((V_S)\) plane. For EHC patterns, noise has a stabilizing effect at \(0 < V_N < 8 \text{V}\), so that the threshold \(V_{S,\text{EHC}}(V_N)\) is larger than \(V_{S,\text{EHC}}(V_N = 0); \text{i.e., in the absence of noise}\), while it is destabilizing at \(V_N > 8 \text{V}\), so that \(V_{S,\text{EHC}}\) decreases with increasing \(V_N\). On the other hand, the threshold \(V_{S,\text{FT}}\) for FT monotonically decreases with increasing \(V_N\), since noise simply destabilizes the initial homeotropic state. (c) \(V_N\)-dependence of thresholds \(V_{S,\text{EHC}}(f_c = 500 \text{Hz})\) for colored noise, and \(V_{S,\text{FT}}\) for white noise and \(V_{S,\text{FT}}\). The noise effect on the system is quite different, depending on \(f_c\) (or \(\tau_N\)). See text for details.

Here let us consider what causes the different behavior between the noise-induced FT and EHC instabilities. This question is critical to understanding the pattern-forming mechanisms by noise. A fundamental difference between the two cases is the following; EHC is a phenomenon far from

\[\text{FT} \quad \text{EHC}\]


In summary, we demonstrate different examples (EHC, SP, and FT) for pure noise-induced bifurcation phenomena. Their corresponding responses to the noise are determined in accordance with flows and without flows, that is, whether the energy dissipation plays an important role or not. Moreover, the specifics (e.g., colored or white) of the noise related to the appearance of EHC and SP depend on the dissipative (flow) types (i.e., small-scale vortices in the $xz$-plane for EHC and large-scale flow in the $xy$-plane for SP). The pure noise-induced EHC depends on the director field slightly below the threshold of convection. Contrarily to the planar case having no tiling of the director, for which the instability is not observed, EHC (and SP) are observed in the homeotropic case having tiling of the director given by the FT. Investigations of the details of patterns and their thresholds in relation to different aspects of noise (e.g., field distribution, spatial correlation) will give us a more fruitful perspective on the noise-induced phenomena, and also noise-controlled applications (i.e., systems in nanotechnology and neuroscience) where noise is unavoidable.

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